## 2.1 Chain Rule: Quiz A Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_quiz.py`). All problems are original; provenance is in `PROVENANCE_QA.md`. To the extent possible under law, the author(s) have waived all copyright and related or neighboring rights to this key. CC0 1.0 Universal: `https://creativecommons.org/publicdomain/zero/1.0/`. Prepared with assistance from Claude, an AI model made by Anthropic.

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## General grading rules

- **Show work.** A bare correct answer earns at most half credit, except where a row says otherwise.
- **Any valid method.** Expanding first, rewriting a quotient with a negative exponent, the quotient rule, the product rule, simplifying an identity first, or carrying the chain rule all the way through are all accepted; no method is required, and long but correct work gets full credit.
- **Notation.** $f'(x)$, $y'$, $\frac{dy}{dx}$ and $\frac{d}{dx}[\ ]$ are all accepted in work. The key writes the chain rule as in the video: $\frac{d}{dx}[A(B(x))] \leftrightarrow \left(\frac{d}{ds}[A(s)]\right)\Big|_{s=B(x)} \cdot \frac{d}{dx}[B(x)]$, with the evaluation bar applied only after the $\frac{d}{ds}$ is gone.
- **Value vs. derivative.** A function value where a derivative was asked (or the reverse) earns 0 for that part.
- **Simplifying.** Answers need not be simplified; equivalent forms are correct.
- **Excluded values.** Where a formula or a method fails at some values of the variable, the key names them. No points are deducted for omitting statements of excluded values.

## Quiz A

| Item | Answer | Rubric / trap |
|---|---|---|
| 1(a) | $4(x^2-3x+1)^3(2x-3)$ | no earlier rule matches the whole power (its base is not $x$), so the chain rule with $A(s)=s^4$, $B(x)=x^2-3x+1$: $\left(4s^3\right)\big\|_{s=x^2-3x+1} \cdot (2x-3)$. Expanding first also works. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* power-rule bait: $4(x^2-3x+1)^3$ alone drops $B'(x)=2x-3$ and earns 0. |
| 1(b) | $(2x-4)\sin(4x-x^2)$ | chain rule, $A(s)=\cos s$, $B(x)=4x-x^2$: $(-\sin s)\big\|_{s=4x-x^2} \cdot (4-2x) = -(4-2x)\sin(4x-x^2)$. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* two signs: the Cosine Rule gives $-\sin$, and $B'(x)=4-2x$; $-\sin(4x-x^2)$ alone, or $-(4-2x)\cos(\ldots)$, earns 0. |
| 1(c) | $3\tan^2 x \sec^2 x$ | $\tan^3 x = (\tan x)^3$; chain rule, $A(s)=s^3$, $B(x)=\tan x$: $\left(3s^2\right)\big\|_{s=\tan x} \cdot \sec^2 x$. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* exponent placement: the outer function is the cube; $3\tan^2 x$ alone, or $\sec^2(x^3)$-type answers, earn 0. |
| 1(d) | $\dfrac{3x^2+6}{2\sqrt{x^3+6x}}$ | rewrite as $(x^3+6x)^{1/2}$ first; chain rule, $A(s)=s^{1/2}$, $B(x)=x^3+6x$: $\left(\frac{1}{2}s^{-1/2}\right)\big\|_{s=x^3+6x} \cdot (3x^2+6)$. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* power-rule bait on the radical: $\frac{1}{2}(x^3+6x)^{-1/2}$ alone earns 0. |

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|---|---|---|
| 1(e) | $(2t-5)^3 + 6t(2t-5)^2$ | product rule; inside it, $\frac{d}{dt}\left[(2t-5)^3\right] = \left(3s^2\right)\big\|_{s=2t-5} \cdot 2 = 6(2t-5)^2$. So $1 \cdot (2t-5)^3 + t \cdot 6(2t-5)^2$, equivalently $(2t-5)^2(8t-5)$. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* $3t(2t-5)^2$ in the second term drops the inner 2; multiplying derivatives, $1 \cdot 6(2t-5)^2$, earns 0. |
| 1(f) | $\dfrac{24}{(1-2x)^3}$ | rewrite as $6(1-2x)^{-2}$; constant multiple, then chain rule, $A(s) = s^{-2}$, $B(x) = 1-2x$: $6 \cdot \left(-2s^{-3}\right)\big\|_{s=1-2x} \cdot (-2)$. The quotient rule also works. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* $B'(x) = -2$: dropping it gives $-\dfrac{12}{(1-2x)^3}$ (wrong sign and size). |
| 2(a) | 0 | $\sin^2(4x) + \cos^2(4x) = 1$, so $f(x) = 6^3 = 216$ for every $x$. The chain-rule route ends in $3(\ldots)^2 \cdot [8\sin(4x)\cos(4x) - 8\cos(4x)\sin(4x)] = 0$. 2 pts, all or nothing for the value 0; work by either route. *Trap:* a constant in disguise; a nonzero answer comes from a slip in the chain-rule route, most often the sign of the derivative of $\cos^2(4x)$ or a dropped inner 4. |
| 2(b) | 0 | $(t+3)^2 - t(t+6) = t^2 + 6t + 9 - t^2 - 6t = 9$, so $p(t) = 3$ for every $t$. The chain-rule route ends in $\frac{1}{2}(9)^{-1/2} \cdot [2(t+3) - (2t+6)] = 0$. 2 pts, all or nothing for the value 0; work by either route. *Trap:* a constant in disguise; any $t$ left in the answer earns 0. |
| 2(c) | $\cos\left(x^5 - 4\sqrt{x} + \sec(3x)\right)$ | the derivative is with respect to $x^5 - 4\sqrt{x} + \sec(3x)$, not $x$. Let $s = x^5 - 4\sqrt{x} + \sec(3x)$; the bracket is $\sin s$, so the derivative is $(\cos s)\big\|_{s=x^5-4\sqrt{x}+\sec(3x)} \cdot$ Dividing $\frac{d}{dx}[\ldots]$ by $\frac{d}{dx}\left[x^5 - 4\sqrt{x} + \sec(3x)\right]$ also works wherever that inner derivative exists and is nonzero (the substitution route has no such gap). Excluded values: $x < 0$, and $x$ where $3x$ is an odd multiple of $\frac{\pi}{2}$; the division route also excludes $x = 0$ and every $x$ where the inner derivative is 0 (infinitely many, since its trig term takes every large value on each period). 2 pts, all or nothing; work by either route. *Trap:* differentiating with respect to $x$: $\cos(\ldots)\left(5x^4 - 2x^{-1/2} + 3\sec(3x)\tan(3x)\right)$ earns 0. |
| 3(a) | 1 | $h(x) = \left(\sqrt[3]{x+4} - 1 + 1\right)^3 - 4 = (x+4) - 4 = x$, so $h'(x) = 1$ everywhere. Chain route: $g(4) = 1$, $f'(x) = 3(x+1)^2$, $g'(x) = \frac{1}{3}(x+4)^{-2/3}$, so $h'(4) = f'(1)\,g'(4) = 12 \cdot \frac{1}{12}$. 2 pts, all or nothing. *Trap:* not noticing $h(x) = x$ costs time, not points; $f'(4)$ or $f'(4)\,g'(4)$ earns 0. |
| 3(b) | 1 | $k(x) = \sqrt[3]{(x+1)^3 - 4 + 4} - 1 = (x+1) - 1 = x$, so $k'(x) = 1$ for every $x$. Chain route: $g'(f(x))\,f'(x) = \frac{1}{3}\left[(x+1)^3\right]^{-2/3} \cdot 3(x+1)^2 = 1$ (for $x \neq -1$; the identity covers $x = -1$). 2 pts. *Trap:* an unsimplified chain-rule expression equivalent to 1 earns full credit; $g'(x)f'(x)$ earns 0. |
| 4(a) | $15x^4\dfrac{dx}{dy} + 4$ | constant multiple, then the chain rule on $x^5$ with $A(s) = s^5$ and $B(y) = x$: $3 \cdot \left(5s^4\right)\big\|_{s=x} \cdot \dfrac{dx}{dy}$; $\dfrac{d}{dy}[4y] = 4$. 2 pts as the trap note says. *Trap:* treating $x$ as the variable: $15x^4 + 4$ earns 0; $15x^4\dfrac{dx}{dy}$ alone (dropping 4) earns 1. |
| 4(b) | $-2x\sin(x^2)\dfrac{dx}{dy}$ | two chain layers: $A(s) = \cos s$, $B(y) = x^2$, and $\dfrac{d}{dy}[x^2] = 2x\dfrac{dx}{dy}$: $(-\sin s)\big\|_{s=x^2} \cdot 2x\dfrac{dx}{dy}$. 2 pts as the trap note says. *Trap:* $-2x\sin(x^2)$ (no $\dfrac{dx}{dy}$) earns 0; $-\sin(x^2)\dfrac{dx}{dy}$ earns 1. |
| 5(a) | 4 | $g(-3) = 1$, $g'(-3) = 2$ (slope of the segment containing $x = -3$); $f'(1) = 2$. $h'(-3) = f'(g(-3))\,g'(-3) = (2)(2)$. 1 pt inner value and slope, 1 pt outer slope at the right $x$, 1 pt value. *Trap:* $f'(-3)\,g'(-3) = 2$ (outer slope read at $-3$ instead of at $g(-3) = 1$); $f'(1) = 2$ alone (inner slope dropped). |

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| 5(b) | $-2$ | $f(1) = -1$, $f'(1) = 2$ (slope of the segment containing $x = 1$); $g'(-1) = -1$. $k'(1) = g'(f(1))\, f'(1) = (-1)(2)$. 1 pt inner value and slope, 1 pt outer slope at the right $x$, 1 pt value. *Trap:* $g'(1)\, f'(1) = 2$ (outer slope read at 1 instead of at $f(1) = -1$); building $f(g(x))$ instead gives $f'(g(1))\, g'(1) = f'(2)\, g'(1) = 2$. |
| 6(a) | $-8$ | $h'(1) = f'(g(1))\, g'(1) = f'(3) \cdot 4 = (-2)(4)$. 1 pt $g(1) = 3$, 1 pt $f'(3)$ and $g'(1)$, 1 pt value. *Trap:* $f'(1)\, g'(1) = 12$ reads the outer slope in the wrong row. Columns run $x$, $g'(x)$, $f(x)$, $g(x)$, $f'(x)$. |
| 6(b) | 15 | $k'(2) = 3[g(2)]^2\, g'(2) = 3(-1)^2(5)$. 1 pt chain rule, 1 pt entries, 1 pt value. *Trap:* $3[g(2)]^2 = 3$ drops $g'(2)$; $3[g'(2)]^2 = 75$ uses the wrong column. |
| 6(c) | 4 | $h(1) = f(g(1)) = f(3)$. 2 pts, all or nothing. *Trap:* this part asks for a value of $h$, not $h'$: $-8$ earns 0; $f(1) = 2$ earns 0. |

Points: $12 + 6 + 4 + 4 + 6 + 8 = 40$.